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ANALYSIS OF THE EFFECTS OF POSITIVE FEEDBACK IN A MULTI-LOOP TRANSISTOR AMPLIFIER

by
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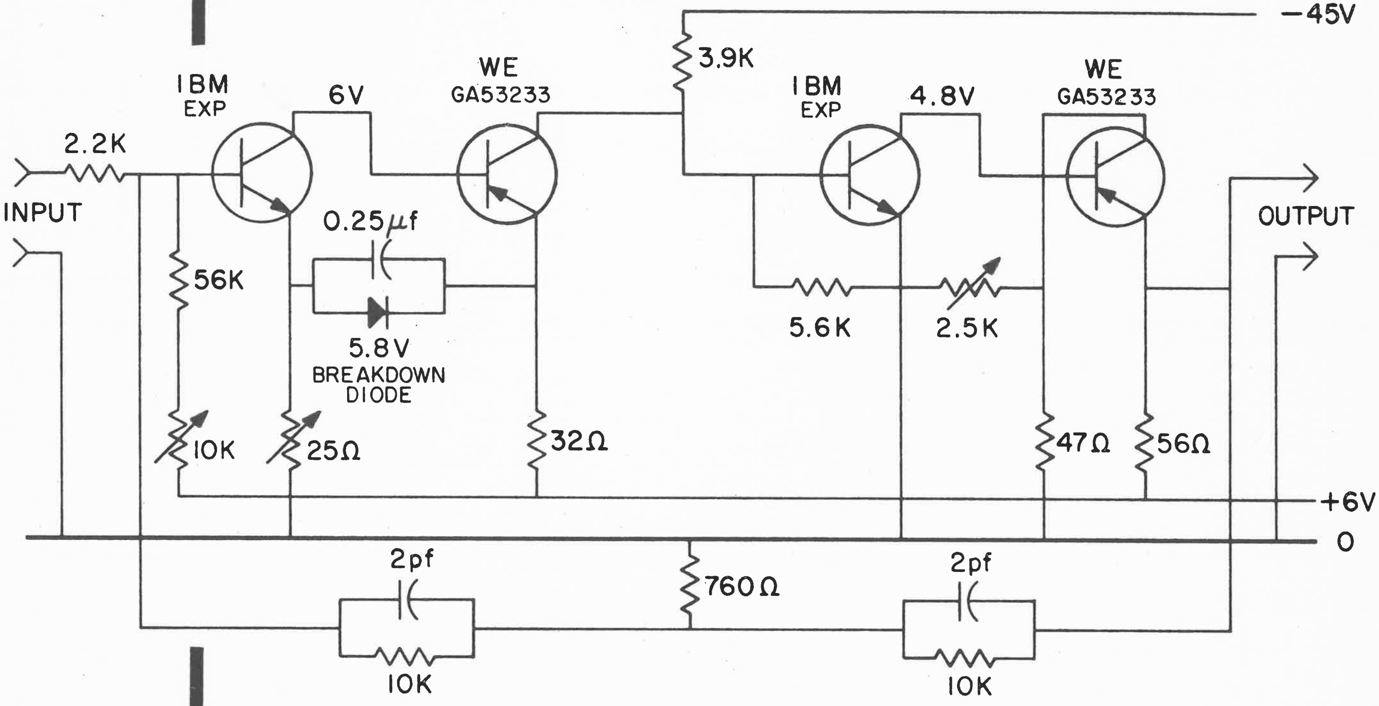

This Circuit was Designed by E. M. Davis
and it was discussed at WESCON in 1958

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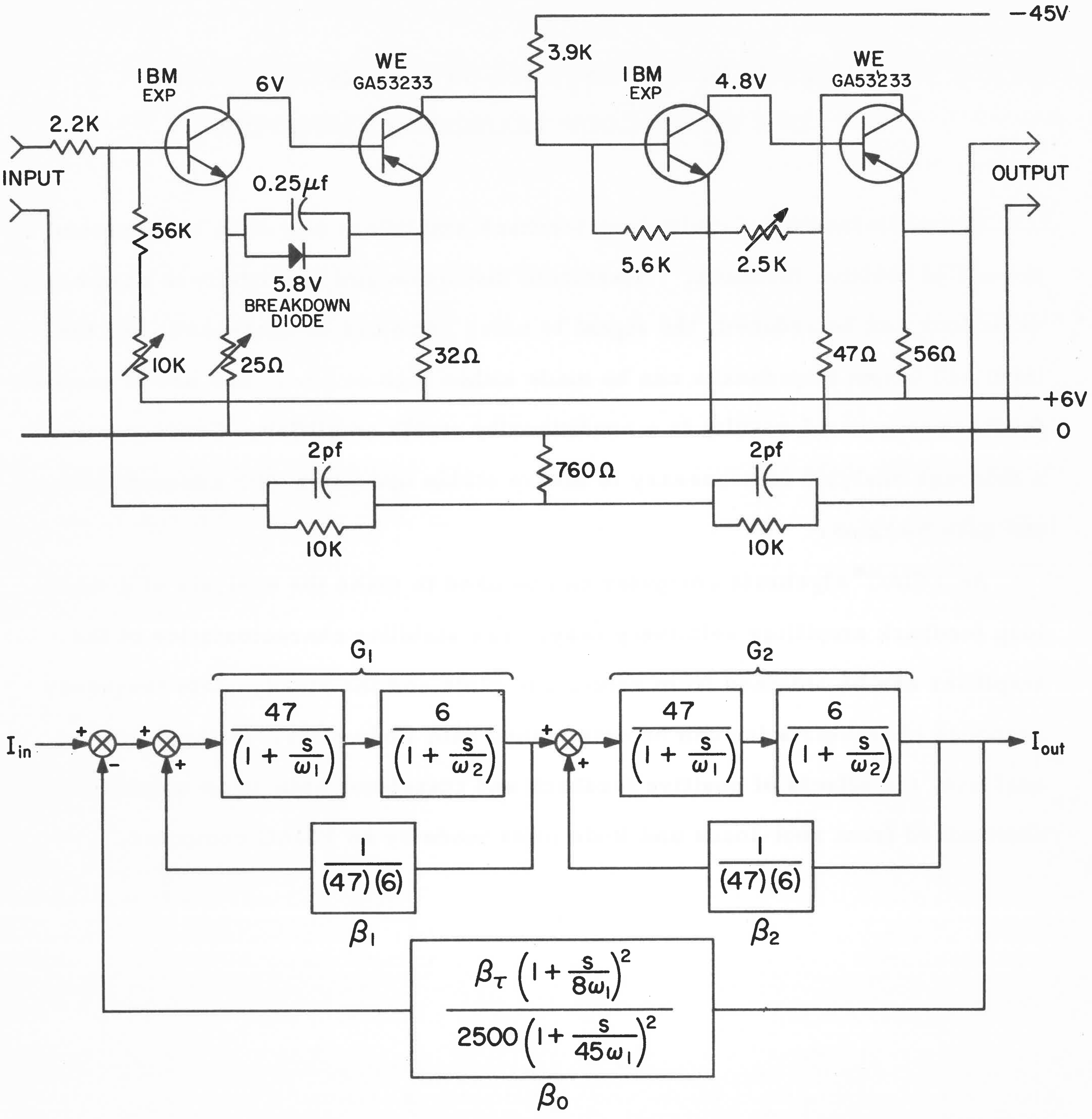
ANALYSIS OF THE EFFECTS OF POSITIVE FEEDBACK IN A MULTI-LOOP TRANSISTOR AMPLIFIER

The performance of multi-loop feedback amplifiers can often be improved by the use of positive feedback.[1] Harmonic distortion and sensitivity to parameter variations can be reduced, the signal to noise ratio can be increased, and the input and output impedances can be made either high or low. The use of positive feedback in general results in a conditionally stable amplifier. Consequently, a thorough analysis is necessary to insure stable operation with adequate phase and gain margins.

An ESIAC® algebraic computer can be used to make the analysis of a multi-loop feedback amplifier relatively easy. The stability characteristics of the amplifier can be obtained from root-locus plots and the steady-state frequency response characteristics can be determined with Bode plots. In the following analysis, the effects of positive feedback and transistor gain in an amplifier are determined from root-locus and Bode plots made by an ESIAC computer.



ω_1 = 6.28 • megarad/sec

ω_2 = 358.00 • megarad/sec

$8\omega_1$ = 50.00 • megarad/sec

$45\omega_1$ = 283.00 • megarad/sec

FIGURE 1. DIRECT COUPLED TRANSISTOR AMPLIFIER.

ANALYZING THE AMPLIFIER

Figure 1 shows the circuit diagram and block diagram of a four-stage, multiloop feedback, transistor current amplifier. The interesting feature of this amplifier is that it has two internal positive feedback loops as well as one overall negative feedback loop.

With Positive Feedback

The transfer functions of the individual stages are shown in the blocks. A transfer function is defined as the ratio of an output quantity to an input quantity. The overall current transfer function of the amplifier from the input to the output, $\frac{I_{out}}{I_{in}}$, is found by repeated application of the familiar feedback equation[2] $\frac{G}{1+\beta G}$ where G is the transfer function of a feedforward branch and β is the transfer function of a feedback branch. The overall closed-loop transfer function of the amplifier is then given by:

$$\frac{I_{out}}{I_{in}} = \frac{\left(\frac{G_1}{1+\beta_1 G_1}\right)\left(\frac{G_2}{1+\beta_2 G_2}\right)}{1+\beta_0\left(\frac{G_1}{1+\beta_1 G_1}\right)\left(\frac{G_2}{1+\beta_2 G_2}\right)} \tag{1}$$

which simplifies to

$$\frac{I_{out}}{I_{in}} = \frac{G_1 G_2}{(1+\beta_1 G_1)(1+\beta_2 G_2) + G_1 G_2 \beta_0} \tag{2}$$

After inserting numerical values and simplifying, Equation (2) becomes

$$\frac{I_{out}}{I_{in}} = \frac{(282)^2}{\left[\left(1+\frac{s}{6.28}\right)\left(1+\frac{s}{358}\right)-1\right]^2 + \frac{\beta_\tau (282)^2\left(1+\frac{s}{50}\right)^2}{2500\left(1+\frac{s}{283}\right)^2}} \tag{3}$$

using units of megaradians per second for s and ω.

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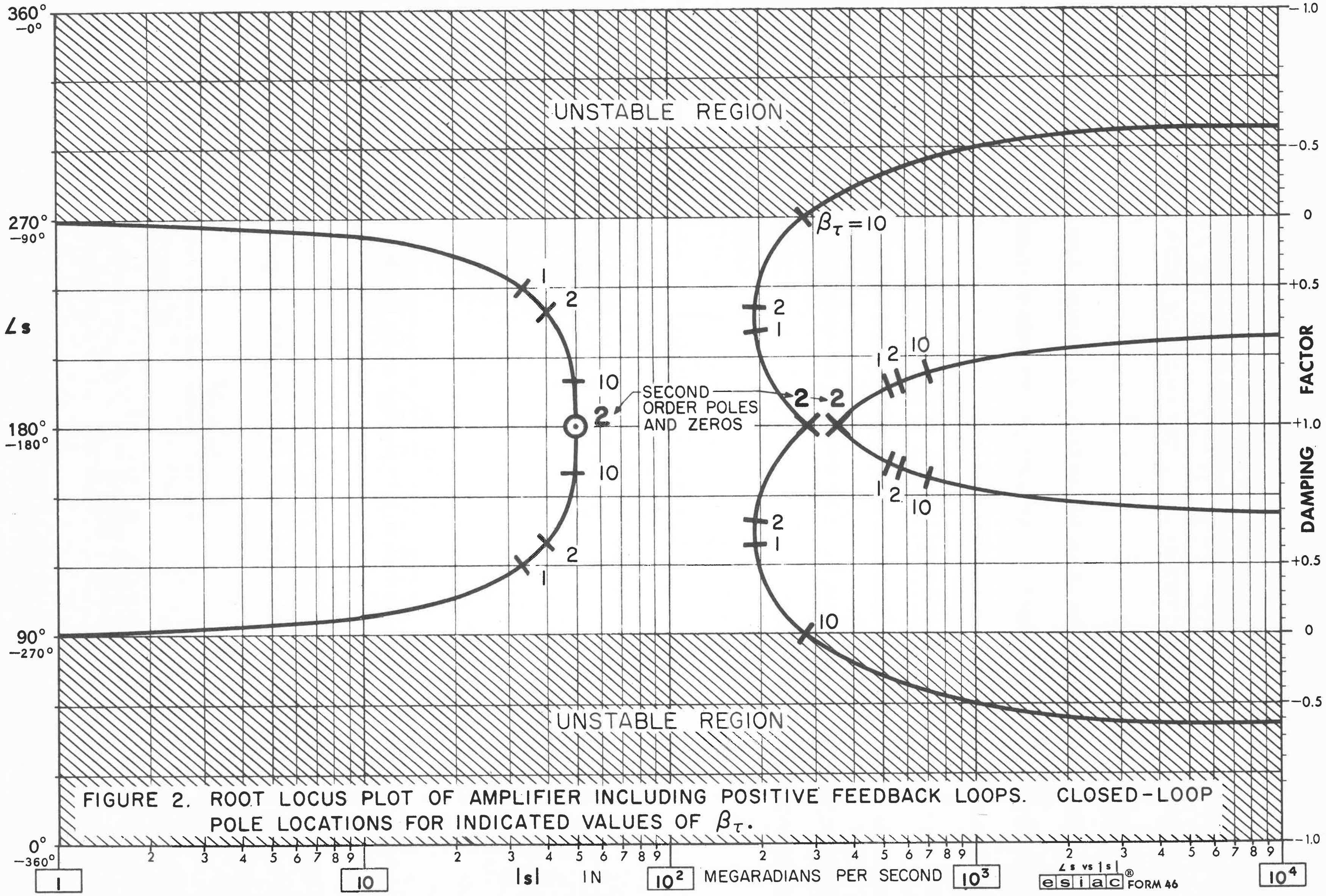

FIGURE 2. ROOT LOCUS PLOT OF AMPLIFIER INCLUDING POSITIVE FEEDBACK LOOPS. CLOSED-LOOP POLE LOCATIONS FOR INDICATED VALUES OF β_τ.

The characteristics of this amplifier such as stability, damping, and frequency response are completely specified by the poles and zeros of its transfer function. The poles and zeros are easily recognized if the transfer function is put in the form of a ratio of factored polynomials. Then the zeros are the values of s that make the individual numerator factors equal to zero and the poles are the values of s that make the individual denominator factors equal to zero.

The problem now is to put Equation (3) in the form of a ratio of factored polynomials. To begin, the first term in the denominator can be factored by multiplying and collecting terms to give

$$\left(1+\frac{s}{6.28}\right)\left(1+\frac{s}{358}\right)-1 = \frac{1}{6.2}\, s\left(1+\frac{s}{365}\right) \tag{4}$$

Then, substituting it into the transfer function gives

$$\frac{I_{out}}{I_{in}} = \frac{(282)^2}{\left(\frac{1}{6.2}\right)^2 s^2\left(1+\frac{s}{365}\right)^2 + \beta_\tau \frac{(282)^2\left(1+\frac{s}{50}\right)^2}{2500\left(1+\frac{s}{283}\right)^2}} \tag{5}$$

or,

$$\frac{I_{out}}{I_{in}} = \frac{(282)^2\ 2500\left(1+\frac{s}{283}\right)^2}{\frac{2500}{(6.2)^2}\, s^2\left(1+\frac{s}{365}\right)^2\left(1+\frac{s}{283}\right)^2 + \beta_\tau(282)^2\left(1+\frac{s}{50}\right)^2} \tag{6}$$

Now solve for the roots of the denominator by setting it equal to zero and rearranging to give

$$-1 = \frac{\beta_\tau\,(1210)\left(1+\frac{s}{50}\right)^2}{s^2\left(1+\frac{s}{365}\right)^2\left(1+\frac{s}{283}\right)^2} \tag{7}$$

which is the form of the general equation for programing the ESIAC. Figure 2 shows the ln s-plane root locus plot for finding the values of s which satisfy the above equation for various values of β_τ. The plot shows that the amplifier will oscillate if β_τ is larger than 10. A reasonable choice would be $\beta_\tau = 1$. The closed-loop pole locations are then

$$\begin{aligned} s = \; & 36\angle 120^\circ, & 36\angle -120^\circ, \\ & 190\angle 140^\circ, & 190\angle -140^\circ, \\ & 540\angle 165^\circ, & 540\angle -165^\circ \end{aligned}$$

Substituting these factors in Equation (6), the closed-loop transfer function for the amplifier becomes

$$\frac{I_{out}}{I_{in}} = \frac{\cancel{(282)}^2 (2500)\left(1 + \frac{s}{283}\right)^2}{\beta_\tau \cancel{(282)}^2\left(1 - \frac{s}{540\angle 165^\circ}\right)\left(1 - \frac{s}{540\angle -165^\circ}\right)\left(1 - \frac{s}{190\angle 140^\circ}\right)\left(1 - \frac{s}{190\angle -140^\circ}\right)\left(1 - \frac{s}{36\angle 120^\circ}\right)\left(1 - \frac{s}{36\angle -120^\circ}\right)} \tag{8}$$

The ESIAC can plot the magnitude and phase versus log-frequency curves, (Bode Plots) by setting $s = j\omega$. These will show the steady-state frequency response of the amplifier with positive feedback. The curves labeled "With Positive Feedback" in Figures 3 and 4 show these Bode Plots.

Without Positive Feedback

For comparison purposes, the amplifier frequency response without the internal positive feedback loops is determined. The closed-loop transfer function without positive feedback is

$$\frac{I_{out}}{I_{in}} = \frac{G_1 G_2}{1 + G_1 G_2 \beta_0} = \frac{\dfrac{(282)^2}{\left(1 + \frac{s}{\omega_1}\right)^2 \left(1 + \frac{s}{\omega_2}\right)^2}}{1 + \dfrac{(282)^2 \beta_\tau \left(1 + \frac{s}{8\omega_2}\right)^2}{2500\left(1 + \frac{s}{\omega_1}\right)^2\left(1 + \frac{s}{\omega_2}\right)^2\left(1 + \frac{s}{45\omega_1}\right)^2}} \tag{9}$$

where $\beta_1 = \beta_2 = 0$. With numerical values,

$$\frac{I_{out}}{I_{in}} = \frac{(2500)(282)^2\left(1+\frac{s}{283}\right)^2}{(2500)\left(1+\frac{s}{6.28}\right)^2\left(1+\frac{s}{358}\right)^2\left(1+\frac{s}{283}\right)^2 + (282)^2\beta_\tau\left(1+\frac{s}{50}\right)^2} \tag{10}$$

The roots of the denominator of Equation (10) are found with the ESIAC by solving the equation:

$$-1 = \frac{31.8\ \beta_\tau\left(1+\frac{s}{50}\right)^2}{\left(1+\frac{s}{6.28}\right)^2\left(1+\frac{s}{358}\right)^2\left(1+\frac{s}{283}\right)^2} \tag{11}$$

Figure 5 shows the resultant root-locus plot on the ln s-plane. The plot shows that the amplifier will oscillate for $\beta_\tau > 10$. A value of $\beta_\tau = 0.97$ is chosen so that the dc gain of the closed loop amplifier will be the same, both with and without the positive feedback loops. For this value of β_τ, the closed-loop pole locations are found in Figure 5 to be

$$\begin{aligned} s = \;& 32\angle 135°, && 32\angle -135°, \\ & 190\angle 140°, && 190\angle -140°, \\ & 540\angle 165°, && 540\angle -165° \end{aligned}$$

The closed-loop transfer function without positive feedback is then,

$$\frac{I_{out}}{I_{in}} = \frac{2500\left(1+\frac{s}{283}\right)^2}{\left(1-\frac{s}{32\angle 135°}\right)\left(1-\frac{s}{32\angle -135°}\right)\left(1-\frac{s}{190\angle 140°}\right)\left(1-\frac{s}{190\angle -140°}\right)\left(1-\frac{s}{540\angle 165°}\right)\left(1-\frac{s}{540\angle -165°}\right)} \tag{12}$$

For $s = j\omega$, the Bode Plots of the amplifier without positive feedback are plotted with the ESIAC. These curves are shown in Figures 3 and 4 and are labeled "Without Positive Feedback".

Figure 4 shows that a 75% increase in bandwidth at the 3 db point can be obtained by using positive feedback in this amplifier.

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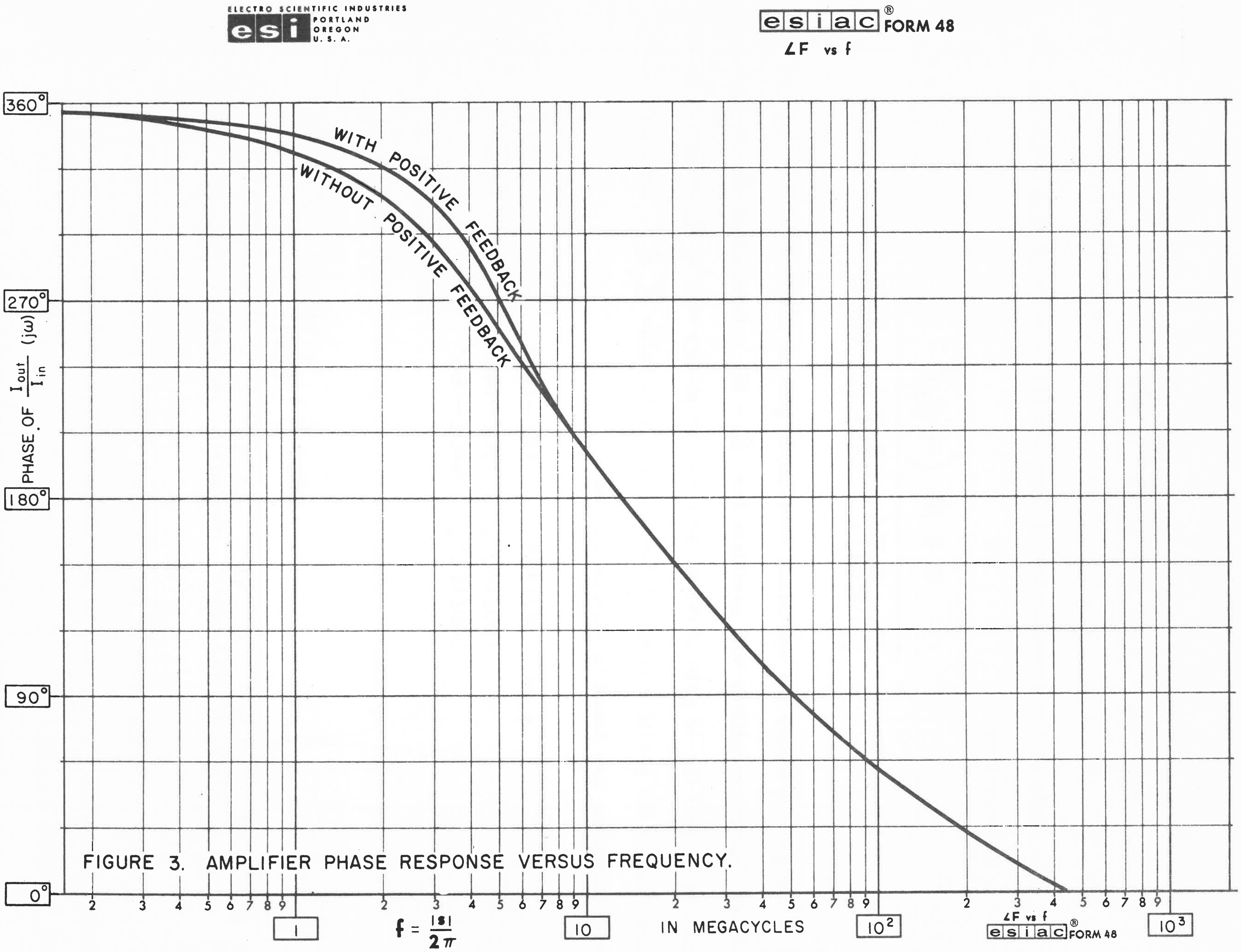

FIGURE 3. AMPLIFIER PHASE RESPONSE VERSUS FREQUENCY.

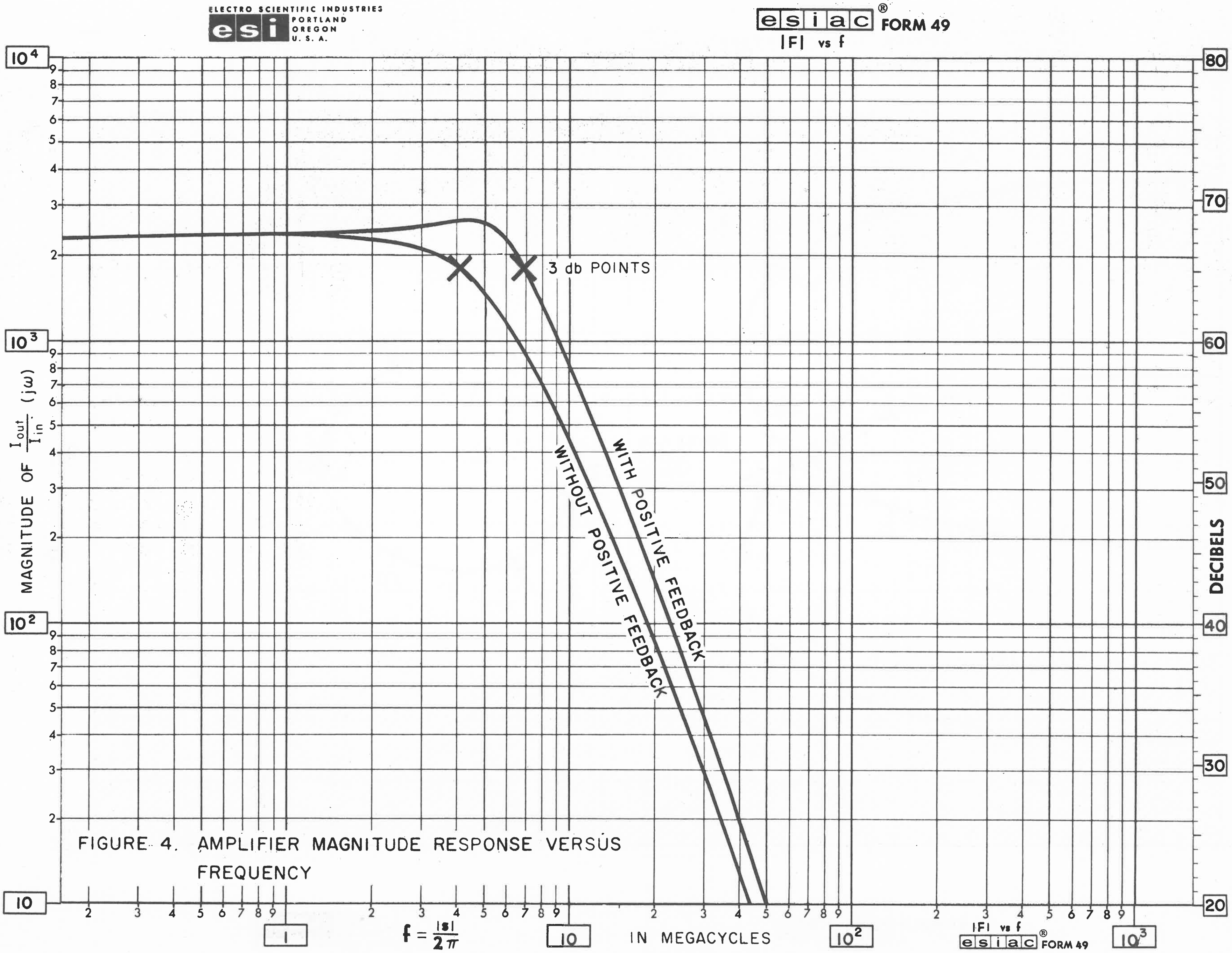

FIGURE 4. AMPLIFIER MAGNITUDE RESPONSE VERSUS FREQUENCY

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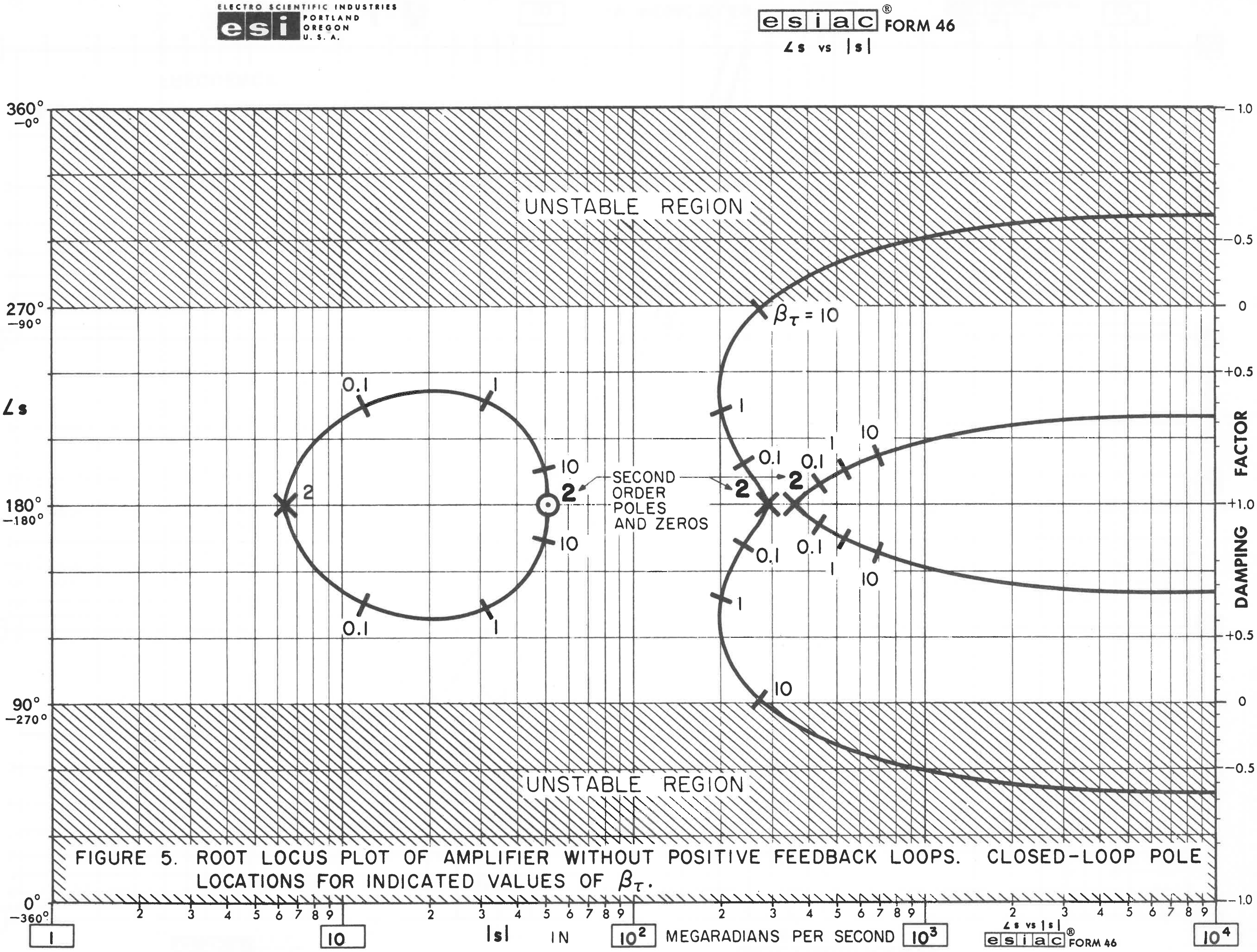

FIGURE 5. ROOT LOCUS PLOT OF AMPLIFIER WITHOUT POSITIVE FEEDBACK LOOPS. CLOSED-LOOP POLE LOCATIONS FOR INDICATED VALUES OF β_τ.

Varying Transistor Gain

Next the effects of a variation in gain of the transistors is investigated. Let A equal the ratio of the gain after variation to the gain before variation. An A = 1.2 would mean that the gain has increased by 20%. Assume also that A is associated with the first stages of the amplifier. The closed-loop transfer function is then

$$\frac{I_{out}}{I_{in}} = \frac{AG_1 G_2}{\left(1+AG_1\beta_1\right)\left(1+G_2\beta_2\right)+AG_1G_2\beta_0} \tag{13}$$

$$= \frac{A\,(282)^2\,(2500)\left(1+\frac{s}{283}\right)^2}{2500\left[\left(1+\frac{s}{6.28}\right)\left(1+\frac{s}{358}\right)-1\right]\left[\left(1+\frac{s}{6.28}\right)\left(1+\frac{s}{358}\right)-A\right]\left(1+\frac{s}{283}\right)^2+A\,(282)^2\beta_\tau\left(1+\frac{s}{50}\right)^2} \tag{14}$$

After factoring the terms in brackets on the left side of the denominator this becomes

$$\frac{I_{out}}{I_{in}} = \frac{A\,(282)^2\,(2500)\left(1+\frac{s}{283}\right)^2}{(1-A)\left(1-\frac{s}{\alpha}\right)\left(1-\frac{s}{\gamma}\right)\left(\frac{s}{6.2}\right)\left(1+\frac{s}{365}\right)\left(1+\frac{s}{283}\right)^2 2500+A\,(282)^2\left(1+\frac{s}{50}\right)^2\beta_\tau} \tag{15}$$

where

$$\alpha = -182 + 1125\sqrt{0.0263-0.0178\,(1-A)}$$

$$\gamma = -182 - 1125\sqrt{0.0263-0.0178\,(1-A)}$$

The closed-loop pole locations are found by solving for the roots of the denominator of Equation (15). Begin by setting the denominator equal to zero to get,

$$\frac{\beta_\tau\left(\frac{A}{1-A}\right)\left(1+\frac{s}{50}\right)^2}{s\left(1-\frac{s}{\alpha}\right)\left(1-\frac{s}{\gamma}\right)\left(1+\frac{s}{365}\right)\left(1+\frac{s}{283}\right)^2} = -5.1 \bullet 10^{-3} \tag{16}$$

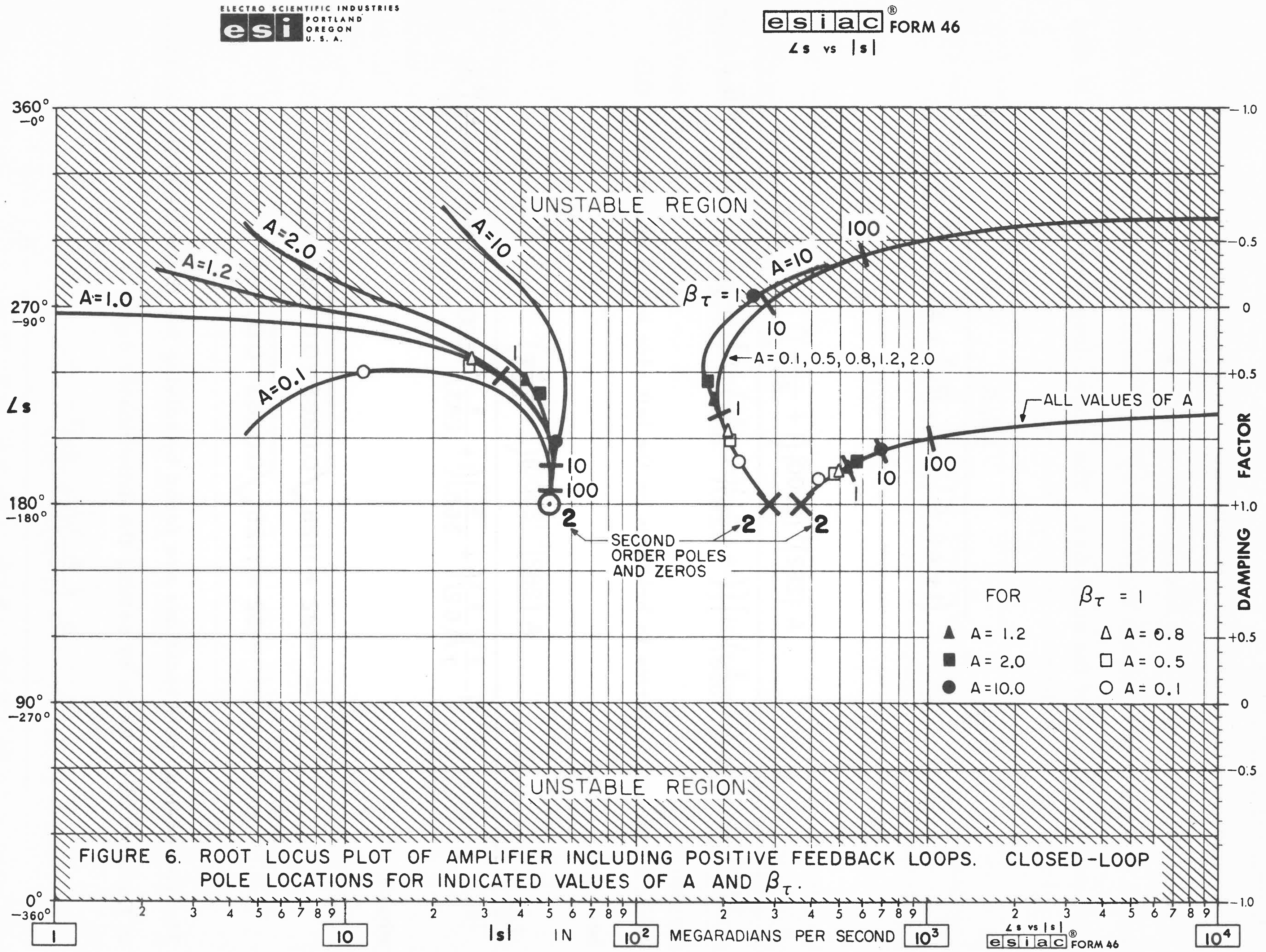

FIGURE 6. ROOT LOCUS PLOT OF AMPLIFIER INCLUDING POSITIVE FEEDBACK LOOPS. CLOSED-LOOP POLE LOCATIONS FOR INDICATED VALUES OF A AND β_τ.

Since α and γ are functions of A, and there are two variables, A and β_τ, the solution of Equation (16) will be a family of curves. These root-locus plots are shown in Figure 6. Also shown in Figure 6 are the closed-loop pole locations for $\beta_\tau = 1$ and various values of A. Notice that, with $\beta_\tau = 1$, the amplifier is stable until A approaches 10. This represents a 20 db increase in open-loop gain. If the open-loop gain is decreased, the amplifier never becomes unstable.

The closed-loop pole locations are easily determined from the curves for any desired value or range of values of A and β_τ. When this is done, the stability characteristics of the amplifier are apparent from Figure 6.

The frequency response curves could then be plotted with the ESIAC to give the steady-state characteristics of the amplifier. If the time-domain transient response is desired, the ESIAC can be used to find the residues at the closed-loop pole locations.[3] After the residues are known, the equation for the time-domain response can be written by using conventional Laplace Transform Techniques.

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3. Hazony, Dov and Riley, Jack, "Evaluating Residues And Coefficients Of High Order Poles," IRE Wescon Convention Record, 1959.